MATHEMATICS

Number Theory	Combinatorics	Geometry
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Algebra	Precalculus	Calculus
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DIVISORS

divisors		
divisors	$m = nq + r$	division
	$\gcd(m, n) = \gcd(r, n)$	Euclidean
	$\gcd(m, n) = am + bn$	Bezout

factorization		
factorization	$n = p_1^{e1} \cdot p_2^{e2}$	fundamental
	$v_p(mn) = v_p(m) + v_p(n)$	exponent
	$t(n) = (e_1 + 1)(e_2 + 1)$	count
	$s(n) = (1 + \ldots p_1^{e1})(1 + \ldots p_2^{e2})$	sum
	$P^2 = n^{t(n)}$	product
phi	$\phi(n) = n \cdot \prod_{i=1}^{k} \left(1 - \frac{1}{p_i}\right)$	count relative primes
	$\phi(mn) = \phi(m) \cdot \phi(n)$	multiplicative

bases		
bases	count	bundles
	base	conversions
unit digit	arithmetic	operation
	only depend	unit digits
decimals	divide ten	shift decimal
	terminating power ten	repeating series

MODULAR ARITHMETIC

congruence		
congruence	$a - b = mk$	definition
	$a \equiv r_a \pmod{m}$	remainder
addition	$a + k \equiv r_a + k \pmod{m}$	constant
	$a + b \equiv r_a + r_b \pmod{m}$	sum
multiplication	$ak \equiv r_a k \pmod{m}$	scale
	$ab \equiv r_a r_b \pmod{m}$	product
	$a^m \equiv r_a^m \pmod{m}$	exponential
division	$ak \equiv bk \pmod{mk}$	common divisor
	$a \equiv b \pmod{\frac{m}{\gcd(k, m)}}$	division
factors	$n \mid m \quad (\text{mod } m) \implies (\text{mod } n)$	divisor
	$(\text{mod } m, n) \implies (\text{mod } mn)$	multiple

divisibility		
divisibility	congruence	relations
	unit digit	two five ten
	last digits	two five ten powers
	sum of digits	three nine
	alternating sum	eleven

LINEAR CONGRUENCE

linear congruence		
linear congruence	$a\,x \equiv b \pmod{m}$	isolate
	multiplication	in lieu division
inverse	$bx \equiv 1 \pmod{m}$	remainder one
	$\gcd(b, m) = 1$	unique inverse
	$3^{-1} = \frac{1}{3}$	reciprocal
system of congruences	first expansion	second substitution
	rearrangement	modulo product

theorems		
Chinese	$x \equiv a_i \pmod{m_i}$	system congruences
	pair wise	relative primes
	$x \equiv c \pmod{m_1 m_2 \ldots}$	exists unique
	specific solution	Euclidean
Fermat	$a^{p-1} \equiv 1 \pmod{p}$	relative prime
	$a^p \equiv a \pmod{p}$	not relative prime
Euler	$a^{\phi(n)} \equiv 1 \pmod{n}$	relative prime
	$a^{\mathrm{lcm}[\phi m, \phi n]} \equiv 1 \pmod{m\,n}$	multiplicative

COUNTING

permutations		
permutations	ordered list	one by one
	$\frac{n!}{(n-r)!}$	multiplication
overcounting	remove	duplicates
	divide by	permutations

combinations		
combinations	unordered list	in or out
	$\binom{n}{r} = \frac{n!}{(n-r)!r!}$	remove duplicates
interpretations	locations	stars and bars
	moves	Pascal's triangle
	coefficients	binomial theorem

identities		
methods	subsets	counting
	paths	counting
	definition	algebra
Pascal's triangle	$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$	Pascal
	$\binom{r}{r} + \ldots + \binom{n}{r} = \binom{n+1}{r+1}$	hockey stick
binomial theorem	$(x+y)^n = \binom{n}{0}x^n + \ldots \binom{n}{n}y^n$	combinations
	$2^n = \binom{n}{0} + \ldots \binom{n}{n}$	one plus one

PROBABILITY

probability		
probability	count events	success total
	addition either	multiplication sequence
compliment	reverse	probability
	shooting stars	hard construction
continuous	non discrete	probability
	overlapping visits	geometric representation
conditional	reduces events	success total
	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$	condition event

expected value		
expected value	weighted average	result probabilities
	requires known	probability distribution
linearity	sum expected values	even if dependent
	optimization with	geometric representation
states	intermediate events	state transitions
	recursion with	values probabilities

SETS

concepts		
PIE	count of union	minimum properties
	alternate sum	over under counting
correspondance	construct matching set	count simpler set
	divide and conquer	equal subsets
partitions	balls and boxes	indistinct boxes
	pigeonhole	min items share property
distributions	balls and boxes	distinct boxes
	correspondance	stars and bars

recursion		
induction	prove statement	numerical or graphical
	positive integer counts	base and inductive
recursion	recurrence relation	closed formula
	golden ratio Fibonnaci	all previous Catalan

generating functions		
generating functions	encapsulate solution space	single event coefficients
	combination representation	product of singles
distributions	single event	power series
	$\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} \binom{n-1+k}{k} x^k$	
partitions	single contribution	exponent
	$\frac{1}{(1-x)(1-x^2)\cdots} = \prod_{k=1}^{\infty} \frac{1}{1-x^k}$	

TRIANGLES

triangles		
similar	proportional	resized smaller larger
	identical	angles side ratios
right	$30° - 60° - 90°$	$1 - \sqrt{3} - 2$
	$45° - 45° - 90°$	$1 - 1 - \sqrt{2}$
area	interior $1/2$ experior -1	Pick
	$\sqrt{s(s-a)(s-b)(s-c)}$	Heron
trigonometry	$\frac{1}{2}ab\sin C$	area
	$\frac{a}{\sin A} = \frac{b}{\sin B} = 2R$	Sine
	$c^2 = a^2 + b^2 - 2ab\cos C$	Cosine

relations		
inequality	two sides	vs third
	$a + b > c$	right triangles
Stewart	arbitrary line	vertex to side
	$b^2m + c^2n = a(d^2 + mn)$	man dad bomb sink
angle bisector	bisector line	divide angle in half
	$\frac{a}{m} = \frac{b}{n}$	ratios
Ceva	arbitrary lines	vertices to sides
	$\frac{a_1}{a_2} \cdot \frac{b_1}{b_2} \cdot \frac{c_1}{c_2} = 1$	one or three
Menelaus	arbitrary external lines	collinear only if
	$\frac{a_1}{a_2} \cdot \frac{b_1}{b_2} \cdot \frac{c_1}{c_2} = 1$	none or two

CENTERS

centers		
circumcenter O	median perpendicular	exterior circle
	equidistant origin vertices	circumradius R
	$\frac{abc}{4R}$	area
incenter I	angle bisector	interior circle
	equidistant origin sides	inradius r
	rs	area
centroid G	median to vertex	six areas four triangles
	one to two	length ratios
orthocenter H	perpendicular to radius	triangle altitudes
	location either	inside or outside
excenter I_A	internal external bisectors	intersection is excenter_A
	equidistant excenter sides	exradius_A

relations		
Euler	$d^2 = R^2 - 2Rr$	distance
	OHG	Euler line
incenter excenter	incenter excenter	all vertices
	reside on	same circle

CIRCLES

inscriptions		
circumference	ratio	diameter angle
	arc length	proportional total
area	limit	right triangles
	sector size	proportional total
inscribed	half of arc	spotlights
	semicircle	right angle
cyclic quadrilateral	opposites supplementary	opposite 90° then cyclic
	$pq = ac + bd$	Ptolemy

intersections		
secants	intersect inside	average arc angles
	intersect outside	half difference arcs
tangents	perpendicular	to radius
	tangent-chord	half chord arc
	tangents meet	equal to point
incircle	$\overline{AT} = s - a$	two tangents meet
	right triangle	hypothenus c
	$\frac{a + b - c}{2}$	inradius
power of a point	line meets circle	at two points
	$OP \cdot OQ = OS \cdot OT$	segment products equal

VECTORS

polygons		
polygons	interior angles	$180^\circ \cdot (n-2)$
	exterior angles	total 360°
	regular polygons	left over triangles
	rs	area
polyhedrons	prisms or cylinders	volume base area height
	base polygon or curved	sides parallelogram or curved
	lateral surface	half perimeter slant height
pointy	three dimensional	pyramid or cone
	base to a point	volume is one third

vectors		
vectors	$\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A}$	addition of reverse
	$\overrightarrow{M} = \frac{\overrightarrow{A} + \overrightarrow{B}}{2}$	midpoint
transformations	translation	addition
	homothety	scale from point
	rotation	complex exponential
distance	$\overrightarrow{A} \cdot \overrightarrow{B} = AB^2$	magnitude
	$ax + by + c = 0$	standardized formula
	$\frac{\lvert ax_0 + by_0 + c \rvert}{\sqrt{a^2 + b^2}}$	point to line
centroid	$G = \left(\frac{a_1 + b_1 + c_1}{3}, \frac{a_2 + b_2 + c_2}{3} \right)$	

FUNCTIONS

functions		
linear	single power combos	slope and intercept
	system of equations	coefficient equality
quadratic	$r+s=-\frac{b}{a}\ \ rs=\frac{c}{a}$	Vietta
	$+\left(\frac{b}{2}\right)^2 \rightarrow \left(x+\frac{b}{2}\right)^2$	complete square
	$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$	formula
absolute	piece wise function	separate to cases
	geometric distance	remove notation squaring
floor	$x=\lfloor x \rfloor + \{x\}$	two parts
	remove floor notation	integer inequality chain

characteristics		
inequality	optimization	constrained and possible
	$\frac{r+s}{2}=-\frac{b}{2a}$	quadratic minimum
inverse	reverse input output	composition is identity
	determine by	reversing variables
symmetry	$f(-x)=f(x)$	even y-axis reflection
	$f(-x)=-f(x)$	odd origin reflection
transformations	addition	shift
	multiplication	scale reflect

SEQUENCES AND SERIES

arithmetic		
sequence	common	difference
	mean has	same difference
series	$\frac{\text{first} + \text{last}}{2} \cdot n$	staircase
	$\frac{2a + d(n-1)}{2} \cdot n$	expansion

geometric		
sequence	common	ratio
	mean has	same ratio
series	$\frac{a(1 - r^n)}{1 - r}$	multiplication
infinite	fractions	repeating decimals
	$\frac{a}{1 - r}$	multiplication

notation		
summation	compact notation	derive closed formula
	$\sum 1$	total count
	$\sum n$	arithmetic series
	$\sum 2^n$	geometric series
linear	$\sum i + \sum j$	addition associative
	$k \sum i$	scaling constant
nested	$\sum \sum (i + j)$	addition distributive
	$\sum \sum ij$	multiplication inner constant

POLYNOMIALS

polynomials		
polynomials	sum of	integer powers
	degree is	highest power
operations	addition	group by degrees
	multiplication	cross expand
degree	product	sum degrees
	zero polynomial	is undefined
zero	factors	are intercepts
	inequalities	by intervals
graphing	intercepts	zero multiplicity
	end behaviour	leading coefficient
	symmetry	$f(-x)$ behaviour
	intervals	by test values

division		
division	$f(x) = g(x) \cdot d(x) + r(x)$	unique pair
	remainder less degree	or zero polynomial
remainder theorem	$f(a) = \dfrac{f(x)}{x - a}$	substitution
	remainder less degree	than linear so is constant
factor theorem	factor and root	equivalence
	$f(x) = a_n(x - r_1)\ldots(x - r_n)$	factored form
fundamental theorem	degree equals	number of complex roots
	search by	iterative factoring

ROOTS

rational		
integer	isolate	constant term
	$r \mid a_0$	divisible
rational	isolate terms	and divide
	$p \mid a_0 \quad q \mid a_n$	divisible
	boundary integers	opposite signs
	search for rational roots	between integers
identity theorem	$n + 1$ roots	zero polynomial
	$n + 1$ equal values	same polynomial

non-rational		
irrational	if real coefficients	then conjugate pair
	show existence	by rational interval
complex	real coefficients	conjugate pair
	imaginary coefficients	convert to real

Vieta		
Vieta	extend from quadratic	to all polynomials
	coefficient equality	of root combinations
	compare polynomial	to its factored form
	symmetric sums	to coefficient ratios
	$p + q + r = -\frac{a_2}{a_3}$	second term
	$pq + qr + rp = \frac{a_1}{a_3}$	third term

IDENTITIES

factoring		
diophantine	problems with	integer solutions
	$xy + ax + by$	$(x + b)(y + a) - ab$
powers difference	$x^n - y^n$	factor obvious
	works for	squares and cubes
powers sum	$x^{2n+1} + y^{2n+1}$	odd alternating signs
	$x^{2n} + y^{2n}$	even complete square
multivariable	$f(h(b), b) = 0$	factor theorem
	$f(a, b) = (a - h(b))\, g(a, b)$	factor zero

identities		
identities	true for	all values
	brute force check	by expansion
ratios	$\frac{a}{b} = \frac{c}{d} = k$	set ratio to variable
	assume is true	and work backwards
	$\frac{a}{b} = \frac{c}{d} = \frac{a + kc}{b + kd}$	equals same ratio
	$\frac{a + kb}{a - kb} = \frac{c + kd}{c - kd}$	equals different ratio
induction	base case identify	induction compels next
	binomial theorem	identity by induction

INEQUALITIES

inequalities		
inequalities	comparison	of two identities
	must be true	for all possible values
	negative product reverses	avoid assume positive

toolkit		
trivial	$x^2 \geq 0$	all real numbers
	$x = 0$	equality case
	chain inequalities	at magic camel value
arithmetic-geometric	$\frac{a+b}{2} \geq \sqrt{ab}$	by trivial inequality
	$a = b$	equality case
	hint for use	sum product
Cauchy-Schwarz	$(a^2+b^2)(c^2+d^2) \geq (ac+bd)^2$	
	$ad = bc$	equality case
	hint for use	sum squares square sums

optimization		
optimization	use	inequalities
	to find	maxima and minima
	show inequality	there is no better value
	show equality	the best value is attainable

RELATIONS

polynomial series		
polynomial series	recurrence relation	closed formula
	telescoping collapses terms	decomposition with zeroes
	$\frac{5-x}{(x-2)(3-x)} = \frac{A}{x-2} + \frac{B}{3-x}$	
arithmetico-geometric	cross product	of all terms
	result approaches	geometric series
finite difference	sequence of	term differences
	degree level	is constant

functional equations		
functional equations	equation input	includes function
	search through	known values
substitution	determine function	constant derives form
	form satisfies function	places restritions on constant
separation	same relation	among variables
	both relations	equal constant
cyclic	repeated identity	step backwards with inverse
	cyclic argument	system of equations

strategies		
substitution	homogeneous	solutions scale
	substitute ratio	or factor
symmetric sums	are elementary	building blocks
	of larger	symmetric polynomials

TRIGONOMETRY

identities		
trigonometry	unit circle projections	periodic sinusoidal
	$x(t) = r \sin \omega t$	general form
identities	$\sin^2 x + \cos^2 x = 1$	Pythagoras
	$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$	
	sum angles	composite triangles
	double expand sums	half substitute doubles
	product to sum	sum to product
geometry	$\frac{1}{2} a b \sin C$	area
	$\frac{a}{\sin A} = \frac{b}{\sin B} = 2R$	Sine
	$c^2 = a^2 + b^2 - 2ab \cos C$	Cosine

parametric		
parametric	intermediate variable	position over time
	alternate representation	simplifies rotations
polar	magnitude angle	Pythagoras
	$x = r \cos \theta$	$y = r \sin \theta$
cylindrical	magnitude angle height	Pythagoras
	(r, θ, z)	height
spherical	magnitude angle angle	Pythagoras
	$x = \rho \sin \phi \cos \theta \quad y = \rho \sin \phi \sin \theta \quad z = \rho \cos \phi$	

COMPLEX

complex numbers		
rectangular	$z = x + yi$	complex plane
	real and imaginary	two components
polar	$z = r(\cos\theta + i\sin\theta)$	substitution conversion
	factor and magnitude	reverse derive angle
exponential	$z = re^{i\theta}$	definition conversion
	simplifies rotations	derives identities

powers		
product	multiplication	magnitudes sum
	$wz = rs(\cos(\alpha+\beta) + i\sin(\alpha+\beta))$	
	$f(x)f(y) = f(x+y)$	functional equation
	$e^{i\theta} = \cos\theta + i\sin\theta$	exponential relation
powers	angle multiples	de Moivre
	$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$	

roots		
unity	$f(x) = x^n - 1$	unsolvable by factoring
	$\omega^n = re^{i\theta} = 1$	exponential form
	$e^{\frac{2\pi}{n}ki}$	roots of unity
general	$z^n = re^{i\theta}$	root of any complex
	$\sqrt[n]{r}\, e^{i\theta/n} \cdot \omega$	scale unity roots
primative	$\omega^3 = 1, \omega^2 \neq 1$	not roots for lessor powers
	ω^k	all roots of unity

VECTORS

dot product		
dot product	map vector to scalar	normal zero
	$\mathbf{a} \cdot \mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$	magnitude
	$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2$	algebra
	$\mathbf{a} \cdot \mathbf{a} = \lvert\mathbf{a}\rvert^2$	substitution
	$\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\lvert\mathbf{a}\rvert^2}\mathbf{a}$	projected vector

cross product		
cross product	map vectors to orthogonal	parallel zero
	$\mathbf{a} \cdot \mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$	magnitude
	$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$	algebra
	$a_1\lvert\text{det}\rvert - a_2\lvert\text{det}\rvert + a_3\lvert\text{det}\rvert$	$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

matrices		
matrices	map vector to vector	unique matrix
	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$	linear equations shorthand
	$\begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_b \end{pmatrix}(\mathbf{b}_1 b_2) = (\mathbf{A}\mathbf{b}_1 \quad \mathbf{A}\mathbf{b}_2)$	product row by column

LIMITS

limits		
limit	approaching	function value
	value at limit	does not matter
	same value	left and right
	trap interval	delta epsilon
rules	linear	combinations
	reciprocal	zero at infinity
	composition	limit of interior
	squeeze theorem	limit of brackets
infinity	zero divide zero	cancel factors
	infinity divide infinity	horizontal asymptote
	constant divide zero	vertical asymptote
	$\lim_{x \to 0} \frac{\sin x}{x} = 1$	squeeze theorem

continuity		
continuity	approaches limit	and actually hits
	does not	skip jump infinite
continuous	polynomials	and combinations
	trigonometric	exponential
theorems	extreme values	continuous has extremes
	intermediate values	continuous passes all values

DIFFERENTIATION

differentiation		
differentiation	rate of change	tangent to curve
	$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	if differentiable then continous
mean value theorem	intermediate value	of continuous interval
	has derivative	of interval slope

rules		
rules	$(fg)' = f'g + fg'$	product
	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$	quotient
	$(g \circ f)' = g'(f)f'$	chain
	$(\sin x)' = \cos x$	trigonometry
	$(e^x)' = e^x$	exponential
	$(\ln x)' = \frac{1}{x}$	logarithmic
inverse	$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$	chain rule
	$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$	inverse rule
	$\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$	inverse trigonometric
implicit	$x^2 + y^2 = 1$	chain rule
	$y' = -\frac{x}{y}$	group differentials
l'Hopital	$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$	zeroes or infinities

INTEGRATION

integration		
integration	area under curve	of a continuous function
	definite integral	limit of Reimann sum
	accumulation of derivatives	with mean value theorem
fundamental theorem	$\int_a^b f(x)\,dx = F(b) - F(a)$	net change
	$F(x) = \int_a^x f(t)dt$	construction

rules		
antiderivative	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	power
	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$	trigonometric
	$\int \frac{1}{x} dx = \ln\lvert x \rvert + C$	logarithmic construction
substitution	$\int g(f(x))f'(x)dx = \int g(u)du$	u-substitution
	$\int u\,dv = uv - \int v\,du$	by parts
	$\frac{5-x}{(x-2)(3-x)} = \frac{A}{x-2} + \frac{B}{3-x}$	partial fractions
parametric	$\int_a^b \sqrt{1+(f'(x))^2}\,dx$	arc length
polar	$A = \int \frac{1}{2} r^2 d\theta$	small angle
improper	$\int_a^\infty f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$	convergence if limit exists

APPROXIMATION

infinite series		
sequence	discrete function	values approach limit
	convergence of limit	monotonic and bounded
series	corresponding sequence	limit of partial sums
	convergence of limit	sequence limit is zero
convergence	divergence	sequence limit not zero
	comparison	to known series
	integral	to improper integral
	limit	of ratio of terms
	ratio	of ratio of successive terms
	alternating	both series convergent

approximation		
approximation	estimate function	near a point
	quadratic match derivatives	solve equal coefficients
	$p(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$	
	error versus actual	bound by next term
Taylor series	$\lim_{n=0}^{\infty} = c_n(x-a)^n$	infinite power series
	converges to function	at all values in interval
	$R = \lim_{n \to \infty} \left\| \frac{c_n}{c_{n+1}} \right\|$	radius of convergence
examples	$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	
	$e^{ix} = \cos x + i \sin x$	Euler

VECTORS

surfaces		
line	$\mathbf{r} = r_0 + t\,\mathbf{v}$	point and direction
	$\mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\,\mathbf{r}_1$	line segment
plane	$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$	point and normal
	$ax + by + cz + d = 0$	standardized form
	$\langle a, b, c \rangle$	normal

functions		
functions	parametric over time	curve through space
	$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$	variable components
curvature	rate of change	tangent through curve
	$\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\lvert \mathbf{r}'(t) \rvert}$ tangent	$\dfrac{ds}{dt} = \lvert \mathbf{r}'(t) \rvert$ arc
	$\kappa = \left\lvert \dfrac{d\mathbf{T}}{ds} \right\rvert$	curvature

multivariable		
differentiation	map vector to scalar	temperature
	$D_u f(x, y) = f_x a + f_y b$	directional derivative
	$\nabla f = \langle f_x, f_y \rangle$	gradient
	$\nabla f = \lambda \nabla g$	Lagrange
integration	nested integration	keep other constant
	$\iint\limits_D f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$	

FIELDS

fields		
fields	map vector to vector	wind direction
	$\nabla f = f_x\,\mathbf{i} + f_y\,\mathbf{j}$	gradient field
conservative	path independent	closed path zeroes
	$\mathbf{F} = \nabla f$	potential function
operator	∇	partial operator
	$\text{div}\,\mathbf{F} = \nabla \cdot \mathbf{F}$	sum partials
	$\text{curl}\,\mathbf{F} = \nabla \times \mathbf{F}$	mix partials

theorems		
fundamental	integrate derivative equals	net change boundaries
	$\int_a^b F'(x)\,dx$	$F(b) - F(a)$
line	curve	points
	$\int_C \nabla f \cdot d\mathbf{r}$	$f(\mathbf{r}(b)) - f(\mathbf{r}(a))$
Green	area	curves
	$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$	$\int_C P\,dx + Q\,dy$
Stokes	surface area	curves
	$\iint_S \text{curl}\,\mathbf{F} \cdot d\mathbf{S}$	$\int_C \mathbf{F} \cdot d\mathbf{r}$
divergence	volume	surface areas
	$\iiint_E \text{div}\,\mathbf{F}\,dV$	$\iint_S \mathbf{F} \cdot d\mathbf{S}$

DIFFERENTIALS

differentials		
differentials	relates variable	with its rate of change
	graphical slope fields	numerical Euler
	separatable	separation of variables

first order		
exponential growth	$y' = ky$	$y = ce^{kx}$
	$\frac{y'}{y} = k$	constant relative rate
logistic growth	$y' = ky - ay^2$	$y = \frac{L}{1 + \alpha e^{-kt}}$
	$\frac{y'}{y} = k - ay$	limiting factor
	$\frac{y'}{y} = k\left(1 - \frac{y}{L}\right)$	$L = \frac{k}{a}$

second order		
second order	$y'' = 0$	$y = c_1x + c_2$
	$y'' - k^2y = 0$	$y = c_1e^{kx} + c_2e^{-kx}$
	$y'' + \omega^2y = 0$	$y = c_1 \sin \omega x + c_2 \cos \omega x$
general form	$y'' + ay' + by = 0$	likely exponentials
	$\lambda^2 + a\lambda + b = 0$	characteristic equation
	$y = c_1e^{r_1x} + c_2e^{r_2x}$	distinct real roots
	$y = e^{rx}(c_1x + c_2)$	double root r
	$y = e^{rx}(c_1 \sin sx + c_2 \cos sx)$	complex roots $r \pm si$